Magnetic field generation in relativistic shocks

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Abstract. We present an analytical estimate for the magnetic field strength generated by the Weibel instability in ultra-relativistic shocks in a hydrogen plasma. We find that the Weibel instability is, by itself, not capable of converting the kinetic energy of protons penetrating the shock front into magnetic field energy. Other (nonlinear) processes must determine the magnetic field strength in the wake of the shock.

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INTRODUCTION

The fireball model for Gamma-ray Bursts (GRBs) explains GRB afterglows as synchrotron radiation coming from the external shocks that are formed when a relativistically expanding fireball (or jet) interacts with surrounding gas [1]. The spectra and luminosity of the observed afterglows indicate a magnetic field strength in the radiating material of about 10% of the equipartition field strength [2]: $B^2/8\pi \sim \varepsilon$ with ε the total post-shock energy density. Such a magnetic field is much stronger than what is expected from simple shock physics: for instance, passive compression of a dynamically unimportant magnetic field in a relativistic shock yields a post-shock field satisfying (for example, see [3]) $B^2/8\pi\varepsilon \sim (v_{\mathrm{A}}/c)^2$, with v_{A} the Alfvén speed *ahead* of the shock, which usually satisfies $v_{\mathrm{A}} \ll c$.

It may be possible that much stronger magnetic fields are generated in the shock transition itself through a Weibel-like instability. This low-frequency electromagnetic beam-instability can develop at the point where the shocked and unshocked plasma penetrate each other. It converts the kinetic energy of the penetrating particle beams into thermal motions and unordered magnetic fields [4]. For external shocks propagating into a hydrogen plasma incoming protons carry most of the kinetic energy in the shock frame. The instability must convert a significant fraction of this energy into magnetic energy in order to reach the required field strength [5]. Here we investigate if this is indeed possible.

SIMPLE COLLISIONLESS SHOCK MODEL

Within the shock transition, where the fireball material encounters the surrounding plasma, the mixing of the shocked (relativistically hot) and the unshocked (cold) plasma produces a plasma with a very anisotropic velocity distribution. As in non-relativistic

collisionless shocks, it is quite likely that a significant fraction of the incoming ions is reflected at the shock transition by a large-scale electrostatic or magnetic field. The resulting situation is known to be unstable [4]: in the so-called Weibel instability the penetrating and reflected particles will bunch together, causing electric currents that induce a magnetic field in the plasma, which in turn causes the particles to bunch even more. If the generated magnetic fields become sufficiently strong they will trap the beam particles, eventually saturating the instability.

Recent numerical simulations [6], [7], [8] show that the electric currents generated by the Weibel instability merge with each other in the wake of the unstable region. This separate process has a longer time-scale, and forms structures on a larger length-scale than the Weibel instability, which occurs mostly on a scale of the order of the plasma skin depth c/ω_{pe}, with ω_{pe} the electron plasma frequency (see below).

In this paper we will concentrate on the proton-driven Weibel instability.

THE WEIBEL INSTABILITY AND ITS SATURATION

Because of the small electron mass, the electron-driven Weibel instability evolves very rapidly [5], with the magnetic field strength growing as $e^{\sigma t}$ with $\sigma \approx \omega_{\text{pb}}$ where $\omega_{\text{pb}} = \sqrt{4\pi e^2 n_{0\text{b}}/m_{\text{e}}}$ is the beam particle plasma frequency based on the proper beam density $n_{0\text{b}}$. When the electron-instability has saturated, the electron velocity distribution will be close to isotropic. The electrons then form a relativistically hot background plasma in which the much slower proton-driven Weibel instability develops because the proton velocity distribution is still very anisotropic (see, for example, figure 6 of [7]). We will investigate the behavior of the protons in this situation.

The main features of the resulting instability can be reproduced by investigating a water-bag proton velocity distribution. We assume two counter-streaming proton beams moving along the x-direction, with a small velocity spread in the z-direction to model thermal motions. The proton momentum $\vec{p}$ is then distributed as:

$$F(\vec{p}) = \frac{n_{\text{p}}}{4p_{z0}}[\delta(p_x - p_{x0}) + \delta(p_x + p_{x0})]\delta(p_y)[\Theta(p_z - p_{z0}) - \Theta(p_z + p_{z0})], \quad (1)$$

where n_{p} is the total proton density, p_{x0} is the beam momentum, p_{z0} is the maximum momentum in the perpendicular direction, $\delta(x)$ is the Dirac delta function and $\Theta(x)$ is the unit step function. We consider the evolution of a wave perturbation with wave vector $\vec{k} = k\vec{e}_z$ and frequency ω. The growth rate of the proton instability is calculated in the usual manner by looking for wave solutions of the linearized equations of motion and Maxwell's equations (for example [9]). The resulting dispersion relation links the complex wave frequency ω to the wave number k. The Weibel instability obeys a dispersion equation of the form (using the notation of [9]):

$$\omega^2 - k^2c^2 + C_{xxz} = 0, \quad (2)$$

where C_{xxz} contains contributions from both the electrons and the protons. Since the electrons are relativistically hot, their contribution is $C_{xxz,\text{e}} = -\omega_{\text{pe}}^2$, with ω_{pe} the effec-
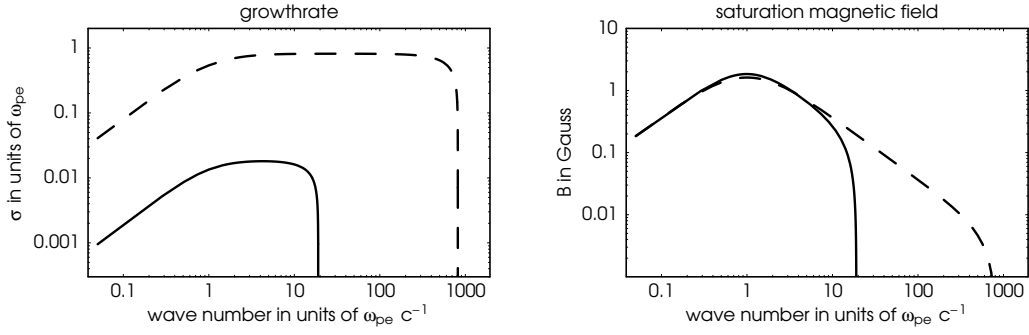
FIGURE 1. Left: growth rate as a function of wave number for a shock with Lorentz factor 1000 and thermal velocity spread $v_{z0} = 0.001$. The solid line is the result for an electron-proton plasma. The dashed line is the result for pure pair plasmas. Right: magnetic field strength as a function of wave number for a shock with the same parameters and $n_p = 2\,\mathrm{cm}^{-3}$.

tive electron plasma frequency which equals

$$\omega_{\mathrm{pe}} = \sqrt{4\pi e^2 n_{\mathrm{e}} / m_{\mathrm{e}} h}\,, \tag{3}$$

where $h \equiv (\varepsilon + P)_{\mathrm{e}} / n_{\mathrm{e}} m_{\mathrm{e}} c^2$ is the electron enthalpy divided by the rest mass energy density, which parameterizes the relativistic mass correction in a relativistically hot plasma.

The proton contribution is [9]:

$$C_{xxz,\mathrm{p}} = \omega_{\mathrm{pi}}^2 \left\{ -\left(\frac{1}{\gamma_b}\right) + \frac{1}{\gamma_{b0}} \frac{u_{x0}^2}{1 + u_{x0}^2} - \frac{1}{\gamma_{b0}} \frac{k^2 v_{x0}^2}{\omega^2 - k^2 v_{z0}^2} \right\}, \tag{4}$$

where $\omega_{\mathrm{pi}} = \sqrt{4\pi e^2 n_{\mathrm{p}} / m_{\mathrm{p}}}$ is the (non-relativistic) proton plasma frequency based on the lab-frame beam density, m_{p} is the proton rest mass, $u_i = p_i/(m_{\mathrm{p}} c)$, $\gamma_{b0} = (1 + u_{x0}^2 + u_{z0}^2)^{1/2}$, $v_i = u_i c / \gamma_{b0}$ and

$$\left(\frac{1}{\gamma_b}\right) = \int \mathrm{d}\vec{p}\, \frac{F(\vec{p})}{\gamma} = \frac{1}{2u_{z0}} \ln\left(\frac{1 + v_{z0}/c}{1 - v_{z0}/c}\right). \tag{5}$$

With these results the dispersion equation (2) becomes a biquadratic equation for ω that has one positive imaginary solution giving the growth rate $\sigma = \mathrm{Im}(\omega)$ of the unstable mode (figure 1, left).

One expects the instability to saturate when the quiver motion of the beam particles in the wave reaches an amplitude Δz such that $k\Delta z \approx 1$. An expression for the corresponding magnetic field strength is derived in [4] and reads in our notation:

$$B_{\mathrm{sat}} = \frac{\gamma_{b0}\, m_{\mathrm{p}}}{v_{x0}\, e} \frac{\sigma^2}{k}. \tag{6}$$

Using the dispersion relation between σ and k we can then find B_{sat} as a function of k (figure 1, right).

A straightforward calculation shows that for small v_{z0} the peak of B_{sat} lies at wave number k_{peak} given by

$$k_{\text{peak}}^2 c^2 \simeq \omega_{\text{pe}}^2 + \omega_{\text{pi}}^2/\gamma_{b0}^3 \, , \qquad (7)$$

with a corresponding growth rate

$$\sigma(k_{\text{peak}}) \simeq \frac{\omega_{\text{pi}}}{\sqrt{2\gamma_{b0}}} \left(\frac{v_{x0}}{c} \right) \, . \qquad (8)$$

The proton-driven Weibel instability occurs for a relatively small range in wavelength compared to the electron-positron case (figure 1): modes with wavelength longer than the electron skin depth ($k < c/\omega_{\text{pe}}$) are inhibited by the response of the background electrons to the proton perturbations. The location where the growth rate levels off corresponds with the location where the saturation magnetic field peaks ($k = k_{\text{peak}}$). If the background electrons had not been as responsive, this location might have been at much lower wave number, and since the wave number is in the denominator of expression (6) for B_{sat}, this would have resulted in a much higher magnetic field. However, we find that the electrons set the location of the peak ($k_{\text{peak}} \simeq \omega_{\text{pe}}/c$) and that the proton Weibel instability can only produce slightly stronger magnetic fields than the electron instability (figure 1, right), despite the larger kinetic energy of the protons.

CONCLUSION

The Weibel instability as it was modeled here is not efficient at converting the kinetic energy of the protons into magnetic fields. However, numerical simulations of collisionless shocks in electron-proton plasmas ([6], [7], [8]) *do* show efficient production of magnetic fields. This can probably be attributed to merging of the electric currents produced by the Weibel instability, *after* the instability that was considered here has stopped. To verify this, the current merging processes will need to be investigated further.

The properties of the magnetic fields generated by these processes will be important for determining the radiation that may be produced in these shocks and that we see in the form of gamma-ray burst afterglows.

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